



CCGPS Frameworks Student Edition

Mathematics

CCGPS Coordinate Algebra Unit 1: Relationships Between Quantities



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"Making Education Work for All Georgians"

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Unit 1
Relationships Between Quantities

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OVERVIEW

In this unit students will:

- interpret units in the context of the problem.
- convert units of measure using dimensional analysis
- when solving a multi-step problem, use units to evaluate the appropriateness of the solution.
- choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- choose and interpret both the scale and the origin in graphs and data displays.
- determine and interpret appropriate quantities when using descriptive modeling.
- determine the accuracy of values based on their limitations in the context of the situation.
- identify the different parts of the expression and explain their meaning within the context of a problem.
- decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.
- create linear and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.
- create equations in two or more variables to represent relationships between quantities.
- graph equations in two variables on a coordinate plane and label the axes and scales.
- write and use a system of equations and/or inequalities to solve a real world problem.
- recognize that the equations and inequalities represent the constraints of the problem.
- solve multi-variable formulas or literal equations, for a specific variable.

The first unit of Coordinate Algebra involves relationships between quantities. Students will be provided with examples of real-world problems that can be modeled by writing an equation or inequality. The tasks begin with simple equations and inequalities and build up to equations in two or more variables. It is important to discuss using appropriate labels and scales on the axes when representing functions with graphs. Students will also explore examples illustrating when it is useful to rewrite a formula by solving for one of the variables in the formula.

In real-world situations, answers are usually represented by numbers associated with units. Units involve measurement and often require a conversion. Measurement involves both precision and accuracy. Estimation and approximation often precede more exact computations. Students need to develop sound mathematical reasoning skills and forms of argument to make reasonable judgments about their solutions. They should be able to decide whether a problem calls for an estimate, for an approximation, or for an exact answer. To accomplish this goal, teachers should provide students with a broad range of contextual problems that offer opportunities for performing operations with quantities involving units. These problems should be connected to science, engineering, economics, finance, medicine, or other career fields.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under “Evidence of Learning” be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

KEY STANDARDS

Reason quantitatively and use units to solve problems.

MCC9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

MCC9-12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

MCC9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpret the structure of expressions

Limit to linear expressions and to exponential expressions with integer exponents.

MCC9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

MCC9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MCC9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.

Create equations that describe numbers or relationships

Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas with a linear focus.

MCC9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear ~~and quadratic functions, and simple rational~~ and exponential functions.

MCC9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

MCC9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

MCC9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

- 1. Make sense of problems and persevere in solving them.** High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
- 2. Reason abstractly and quantitatively.** High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols

involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

- 3. Construct viable arguments and critique the reasoning of others.** High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
- 4. Model with mathematics.** High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
- 5. Use appropriate tools strategically.** High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content

located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

- 6. Attend to precision.** High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
- 7. Look for and make use of structure.** By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y . High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
- 8. Look for and express regularity in repeated reasoning.** High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who do not have an understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify

conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a missing mathematical knowledge effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

ENDURING UNDERSTANDINGS

- Identify the vocabulary for the parts that make up the whole expression. Interpret their meaning in terms of a context.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.
- Select appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Create linear and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.
- Recognize that exponential functions can be used to model situations of growth, including the growth of an investment through compound interest.
- Create equations in two or more variables to represent relationships between quantities.
- Graph equations in two variables on a coordinate plane and label the axes and scales.
- Write and use a system of equations and/or inequalities to solve a real world problem.
- Solve multi-variable formulas or literal equations for a specific variable in a linear expression.

CONCEPTS/SKILLS TO MAINTAIN

Students may not realize the importance of unit conversion in conjunction with computation when solving problems involving measurement. Since today’s calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than is required.

Measuring commonly used objects and choosing proper units for measurement are part of the mathematics curriculum prior to high school. In high school, students experience a broader variety of units through real-world situations and modeling, along with the exploration of the different levels of accuracy and precision of the answers.

An introduction to the use of variable expressions and their meaning, as well as the use of variables and expressions in real-life situations, is included in the Expressions and Equations Domain of Grade 7.

Working with expressions and equations, including formulas, is an integral part of the curriculum in Grades 7 and 8. In high school, students explore in more depth the use of equations and inequalities to model real-world problems, including restricting domains and ranges to fit the problem's context, as well as rewriting formulas for a variable of interest.

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess to determine whether instructional time should be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Using the Pythagorean Theorem
- Understanding slope as a rate of change of one quantity in relation to another quantity
- Interpreting a graph
- Creating a table of values
- Working with functions
- Writing a linear equation
- Using inverse operations to isolate variables and solve equations
- Maintaining order of operations
- Understanding notation for inequalities
- Being able to read and write inequality symbols
- Graphing equations and inequalities on the coordinate plane
- Understanding and using properties of exponents
- Graphing points
- Choosing appropriate scales and labeling a graph

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children.

Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.

<http://www.amathsdictionaryforkids.com/>

This web site has activities to help students more fully understand and retain new vocabulary.

<http://intermath.coe.uga.edu/dictionary/homepg.asp>

Definitions and activities for these and other terms can be found on the Intermath website.

Intermath is geared towards middle and high school students.

- **Algebra:** The branch of mathematics that deals with relationships between numbers, utilizing letters and other symbols to represent specific sets of numbers, or to describe a pattern of relationships between numbers.
- **Coefficient:** A number multiplied by a variable.
- **Domain:** The set of x -coordinates of the set of points on a graph; the set of x -coordinates of a given set of ordered pairs. The value that is the input in a function or relation.
- **Equation:** A number sentence that contains an equals symbol.
- **Expression:** A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.
- **Function:** A rule of matching elements of two sets of numbers in which an input value from the first set has only one output value in the second set.
- **Inequality:** Any mathematical sentence that contains the symbols $>$ (greater than), $<$ (less than), \leq (less than or equal to), or \geq (greater than or equal to).
- **Ordered Pair:** A pair of numbers, (x, y) , that indicate the position of a point on a Cartesian plane.
- **Perimeter:** The sum of the lengths of the sides of a polygon.
- **Pythagorean Theorem:** It is a theorem that states a relationship that exists in any right triangle. If the lengths of the legs in the right triangle are a and b and the length of the hypotenuse is c , we can write the theorem as the following equation:

$$a^2 + b^2 = c^2$$

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- **Range:** The y-coordinates of the set of points on a graph. Also, the y-coordinates of a given set of ordered pairs. The range is the output in a function or a relation.
- **Substitution:** To replace one element of a mathematical equation or expression with another.
- **Variable:** A letter or symbol used to represent a number.

Acting Out

Mathematical Goals

- Model and write an equation in one variable and solve a problem in context.
- Create one-variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

Common Core State Standards

MCC9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational~~ and exponential functions.

MCC9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

MCC9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

MCC9-12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

MCC9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
- 4. Model with mathematics.**
- 5. Use appropriate tools strategically.**
- 6. Attend to precision.**

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Part 1:

Erik and Kim are actors at a theater.

Erik lives 5 miles from the theater and Kim lives 3 miles from the theater.

Their boss, the director, wonders how far apart the actors live.

- On grid paper, pick a point to represent the location of the theater.
- Illustrate all of the possible places that Erik could live on the grid paper.
- Using a different color, illustrate all of the possible places that Kim could live on the grid paper.
- What is the smallest distance, d , that could separate their homes? How did you know?
- What is the largest distance, d , that could separate their homes? How did you know?
- Write and graph an inequality in terms of d to show their boss all of the possible distances that could separate the homes of the 2 actors.

Part 2:

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.

Lucy's Linear Equations and Inequalities

Mathematical Goals

- Create one-variable linear equations and inequalities from contextual situations.
- Solve and interpret the solution to multi-step linear equations and inequalities in context.

Common Core State Standards

MCC9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ exponential functions.

MCC9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
- 4. Model with mathematics.**
- 7. Look for and make use of structure.**

Lucy has been assigned the following linear equations and inequality word problems. Help her solve each problem below by using a five step plan.

- Drawing a Sketch(if necessary)
 - Defining a Variable
 - Setting up an equation or inequality
 - Solve the equation or inequality
 - Make sure you answer the question
1. The sum of 38 and twice a number is 124. Find the number.
 2. The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.
 3. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width.
 4. The length of a rectangle is 4 cm more than the width and the perimeter is at least 48 cm. What are the smallest possible dimensions for the rectangle?

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5. Find three consecutive integers whose sum is 171.
6. Find four consecutive even integers whose sum is 244.
7. Alex has twice as much money as Jennifer. Jennifer has \$6 less than Shannon. Together they have \$54. How much money does each have?
8. There are three exams in a marking period. A student received grades of 75 and 81 on the first two exams. What grade must the student earn on the last exam to get an average of no less than 80 for the marking period?

Forget the Formula

Mathematical Goals

- Rearrange formulas to highlight a quantity of interest.
- Create equations in two variables to represent relationships.
- Understand how the change in one variable affects the other variable in a given situation.
- Write and graph an equation to represent a linear relationship.
- Extend the concepts used in solving numerical equations to rearranging formulas for a particular variable.

Common Core State Standards

MCC9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

MCC9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

MCC9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

MCC9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

MCC9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

MCC9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

MCC9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
- 8. Look for and express regularity in repeated reasoning.**

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Mrs. Howell, your science teacher, overheard two of her students talking about how to convert temperatures from Celsius to Fahrenheit and vice versa. The students said they knew there was a formula, but they did not remember what it was. Mrs. Howell remarked to you that if they just knew about the freezing point and boiling point of water for each temperature scale, the formula could easily be “rediscovered.” Mrs. Howell has asked you to write a written explanation for how to find the formula, showing all your calculations. Mrs. Howell also wants you to include an explanation of each of the parts of the formula. (Don’t forget to include the formula for Celsius to Fahrenheit and the formula for Fahrenheit to Celsius.)

Cara's Candles Revisited

Mathematical Goals

- Determine whether a point is a solution to an equation.
- Determine whether a solution has meaning in a real-world context.
- Interpret whether the solution is viable from a given model.
- Write and graph equations and inequalities representing constraints in contextual situations.

Common Core State Standards

MCC9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

MCC9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

MCC9-12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

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Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
- 3. Construct viable arguments and critique the reasoning of others.**
- 4. Model with mathematics.**
- 7. Look for and make use of structure.**
- 8. Look for and express regularity in repeated reasoning.**

Cara's Candles Revisited

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

Cara started filling out the following table to help determine whether these two candles would ever reach the same height at the same time if allowed to burn the same length of time. Finish the table for Cara. Use the data in the table to determine what time the two candles will be at the same height.

Also, she wants to know what height the two candles would be at that time. If it is not possible, she wants to know why it could not happen and what would need to be true in

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order for them to be able to reach the same height. To help Cara understand what you are doing, justify your results. Explain your thinking using the table and create a graphical representation of the situation.

Time (hours)	16 cm candle height (cm)	12 cm candle height (cm)
0	16	12
1	13.5	10.5
2		
3		
4		
5		
6		
7		

Using the table, write an equation for the height of each candle in terms of the number of hours it has burned. Be sure to include any constraints for the equation.

Cara has another candle that is 15 cm tall. How fast must it burn in order to also be 6 cm tall after 4 hours? Explain your thinking.

If Cara had a candle that burned 3 cm every hour, how tall would it need to be to also reach the same height as the other three candles after 4 hours? Explain your thinking.

The Yo-Yo Problem

Adapted from PBS Mathline: <http://www.pbs.org/teachers/mathline/lessonplans/pdf/hsmp/yoyo.pdf>

Mathematical Goals

- Explore linear patterns.
- Create one variable and two variable linear equations.
- Graph equations on coordinate axes with labels and scales.

Common Core State Standards

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MCC9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

MCC9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MCC9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
- 4. Model with mathematics.**
- 6. Attend to precision.**
- 7. Look for and make use of structure.**

8. Look for and express regularity in repeated reasoning.

Part 1: The Yo-Yo Problem

Andy wants to buy a very special yo-yo. He is hoping to be able to save enough money to buy it in time to take a class in which he would learn how to do many fancy tricks. The 5-ounce aluminum yo-yo costs \$89.99 plus 6% sales tax. Andy has already saved \$17.25, and he is earning \$7.20 a week by doing odd jobs and chores. How many weeks will it take him to save enough money for the yo-yo?

1. How much sales tax will Andy have to pay?
2. What will be the total cost of the yo-yo, including tax?
3. Let w be the number of weeks that it will take Andy to save enough money to buy the yo-yo. Write an algebraic equation that will help you solve the problem.
4. Solve your equation for w , and check your answer. Be prepared to present your solution to the class.



Part 2: The Penny Pattern

5. Create a pattern using pennies. Stage one of the pattern is shown above—one penny surrounded by six additional pennies. To create each additional stage of the design, place more pennies extending out from the six that surround the center penny. Continue making this design until you have used up all of your pennies. On the back of this sheet, sketch the first four stages of the pattern.
6. Using your penny pattern or the sketches of your penny pattern, create a table of values.

Stage Number, n	1	2	3	4	5
Number of Pennies Required					

7. How many pennies are needed to make stage 6, stage 7, and stage 8 of the penny pattern? How did you determine your answer?

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8. Write an algebraic model that expresses the relationship between the stage number, n , and the number of pennies required to make that design, p .
9. Use your model to determine how many pennies are needed to make stage 80, stage 95, and stage 100 of the penny pattern.
10. If you use 127 pennies to make the penny pattern, how many pennies will be in each spoke coming out from the center penny?

Paper Folding

Adapted from PBS Mathline:

<http://www.pbs.org/teachers/mathline/lessonplans/pdf/hsmp/rhinos.pdf>

Mathematical Goals

- Write and graph an equation to represent an exponential relationship.
- Model a data set using an equation.
- Choose the best form of an equation to model exponential functions.
- Use properties of exponents to solve and interpret the solution to exponential equations in context.
- Graph equations on coordinate axes with labels and scales.

Common Core State Standards

MCC9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ exponential functions.

MCC9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

MCC9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

MCC9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

MCC9-12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

MCC9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

MCC9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

MCC9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MCC9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**

- 3. Construct viable arguments and critique the reasoning of others.**
- 4. Model with mathematics.**
- 6. Attend to precision.**
- 7. Look for and make use of structure.**
- 8. Look for and express regularity in repeated reasoning.**

The Paper Folding Activity

Part 1: Number of Sections

Number of Folds	Number of Sections
0	
1	
2	
3	
4	
5	
6	

1. Fold an 8.5 x 11" sheet of paper in half and determine the number of sections the paper has after you have made the fold.
2. Record this data in the table and continue in the same manner until it becomes too hard to fold the paper.
3. Make a scatter plot of your data in a separate sheet of graph paper.
4. Determine a mathematical model that represents this data by examining the patterns in the table.
5. What might be different if you tried this experiment with an 8.5 x 11" sheet of wax paper or tissue paper?

Part 2: Area of Smallest Section

Number of Folds	Area of Smallest Section
0	1
1	
2	
3	
4	
5	
6	

6. Fold an 8.5 x 11" sheet of paper in half and determine the area of the smallest section after you have made the fold.
7. Record this data in the table and continue in the same manner until it becomes too hard to fold the paper.
8. Make a scatter plot of your data on a separate sheet of graph paper.

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9. Determine a mathematical model that represents this data by examining the patterns in the table.